Informative hypotheses evaluation

Bayes factors and information criteria

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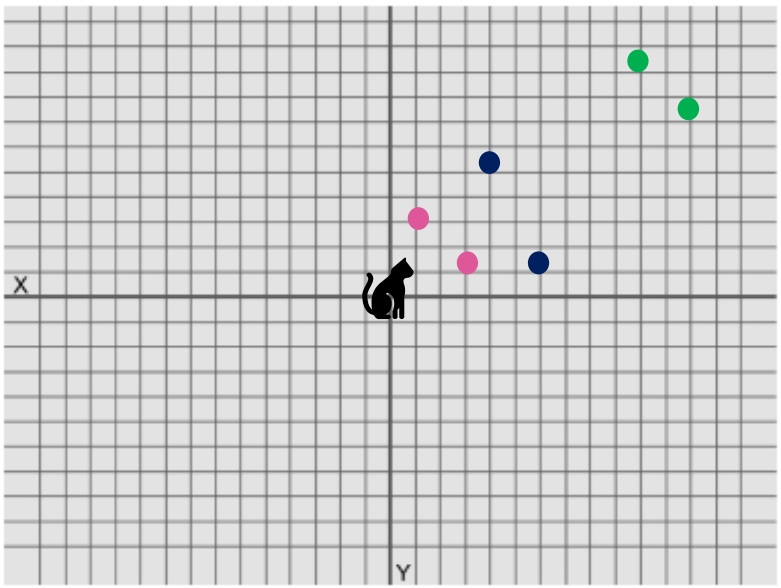
A Research Project and Its Replication

An experiment with three conditions:

- The "close" condition
- The "intermediate" condition
- The "distant" condition



Participants Rated:

Attachment to:

- Siblings
- Parents
- Home-town

on a

1 (not at all strong) – 7 (extremely strong)
Likert scale

which are
averaged to obtain the dependent variable

The description given here is a modification of and inspired by the actual experiment executed by Williams, L.E. and Bargh, J.A. (2008). Keeping One's Distance. The Influence of Spatial Distance Cues on Affect and Evaluation. *Psychological Science, 19,* 302-308.

The Main Research Outcomes

Williams and Bargh (2008) tested:

H_0: $\mu_{close} = \mu_{intermediate} = \mu_{distant}$,

that is, the three means are equal

rendering

p-value = .01, that is, smaller than .05, that is, the means are significantly different

with

$m_{close} = 5.61$, $m_{intermediate} = 5.23$, $m_{distant} = 4.86$

and

$\eta^2 = .11$, that is, the three conditions explain 11% of the variation in attachment, which is a medium to strong effect of condition

The replication by Joy-Gaba, Clay, and Cleary (2016) rendered

p-value = .79

with

$m_{close} = 5.44$, $m_{intermediate} = 5.31$, $m_{distant} = 5.31$

And

$\eta^2 = .00$

Joy-Gaba, J., Clay, R., and Cleary, H. (2016). Replication of keeping one's distance: The influence of spatial distance cues on affect and evaluation by Williams L.E. and Bargh J.A. (2008) *Psychological Science, 19,* 302-308). Retrieved from https://osf.io/a78bm/

The Replication Crisis

The Replication Crisis

This is only one of 100 psychological experiments of which only about 33% were successfully replicated (OSC, 2015).

This resulted in a reduced trust in science by scientists and society: The replication crisis was born.

Scientists are alerted:

- Estimating the reproducibility of psychological science (OSC, 2015)
- An open investigation of the reproducibility of cancer biology research (Errington et al., 2014)

"Society" is alerted:

- Is psychology a real science? (Is psychologie wel een echte wetenschap, Volkskrant, 12-8-2016)
- Public Trust in Science (Rathenau Instituut, August 28, 2018)

Open Science Collaboration. (2015). Estimating the reproducibility of psychological science. *Science, 349*, 6251. https://osf.io/ezcuj/
Errington, T.M., Iorns, E., Gunn, W., Tan, F.E., Lomax, J., and Nosek, B.A. (2014). An open investigation of the reproducibilty of cancer biology research. *eLIFE, 3*, e04333. https://elifesciences.org/collections/9b1e83d1/reproducibility-project-cancer-biology
Volkskrant (2016). https://www.volkskrant.nl/columns-opinie/is-psychologie-wel-een-echte-wetenschap~b9978e6c
Rathenau Instituut (2018). Public Trust in Science. https://www.rathenau.nl/en/science-figures/impact/trust-science/public-trust-science

The p-value and The .05

p-value

The p-value is the probability of the observed data (or data that deviate more from H_0) assuming that H_0 is true.

.05

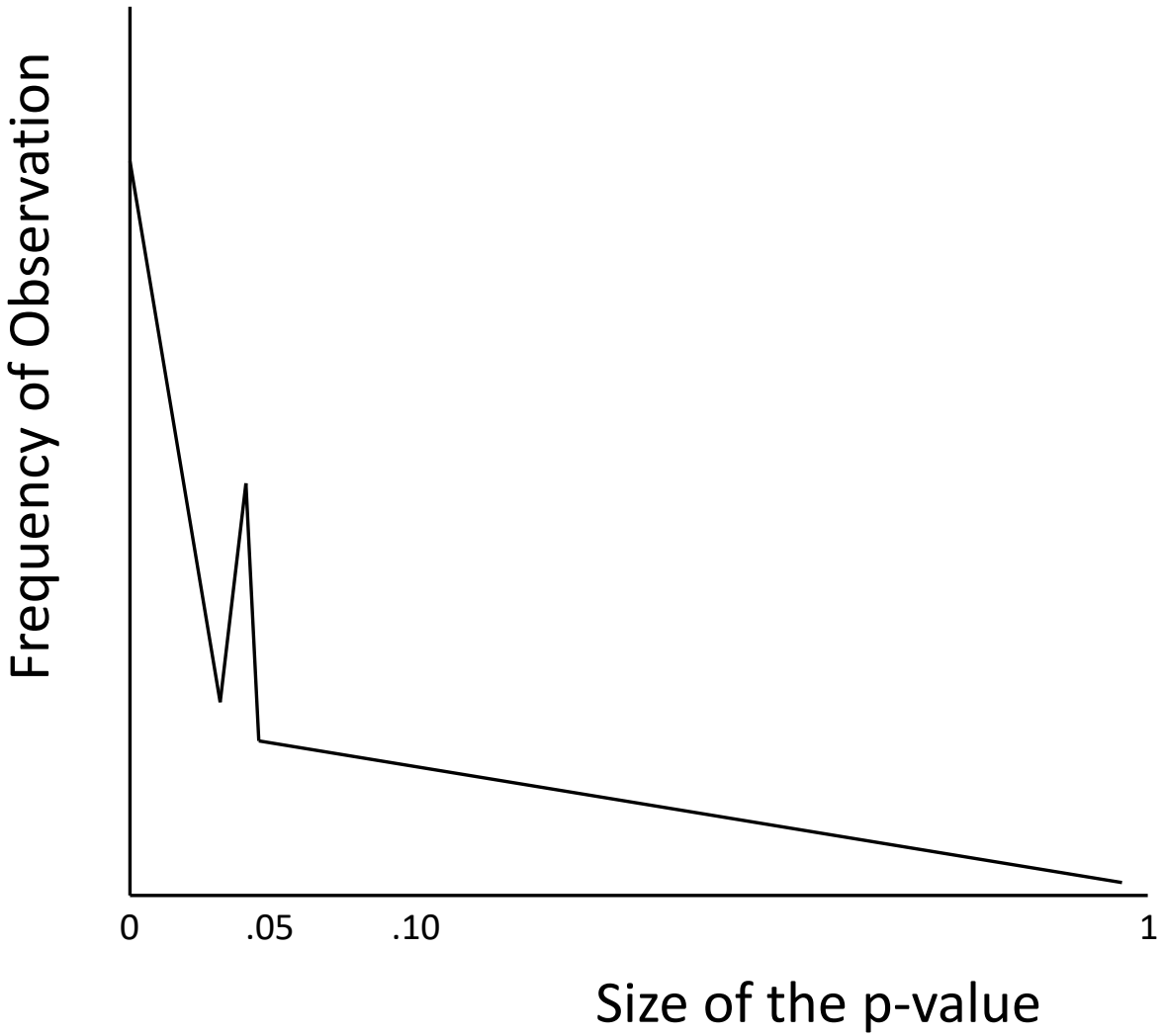
If the p-value is smaller than .05, it is considered to be so small that H_0 has to be rejected.

Causes of the Replication Crisis

Masicampo en Lalande (2012) collected the p-values published in the journals: Psychological Science, Journal of Personality and Social Psychology, and Journal of Experimental Psychology: General.

1. Masicampo, E.J. and Lalande, D.R. (2012). A peculiar prevalence of p values just below .05. *The Quarterly Journal of Experimental Psychology, 65,* 2271-2279.

Causes of the Replication Crisis



Questionable Research Practices

1. After testing the null-hypothesis, the resulting p-value is .06. But after removing three persons with unexpected low scores on the dependent variable, the p-value becomes .04
2. After testing the effect of treatment/control on three operationalisations of depression, resulting in p-values of .04, .12, and .34, only "the significant" p-value is reported
3. Any other examples ...

Incentives for Questionable Research Practices

Found somewhere on the internet:

TS college 8 beamer.pdf

. . . and has real-life consequences

p value scale				
*** .001	very highly significant	there is an effect definitely for sure	elation exuberance smugness	nobel price tenur research grant
** .01	highly significant	there is an effect	great pleasure dancing drinking	phd price top publication
* .05	significant (phew)	most likely there is an effect	relief cheerfulness	consolation price fair publication
? .10	approaching significance	almost probably an effect but low power	frustration if only	counseling stress leave
	nonsignificant	no effect	despair depression	medication reconsider life goals

after Geoff Cumming, 2012

Prevalence of Questionable Research Practices

1. About 2% of scientists admits to having fabricated or falsified research data, or to have altered or modified results to improve the outcome
2. About 33% of scientists admits to having used questionable research practices
3. How about "me" and "you" ...

1. Fanelli, D. (2009). How many scientists fabricate and falsify research? A systematic review and meta-analysis of survey data. *PloS ONE, 4,* e5738.
2. Ioannides, J.P.A. (2005). Why most published research findings are false. *PLoS Medicine, 2*, e124.

Publication Bias

1. In 1981, a psychologist investigated "feeling the future" ... The p-value for "H0: the choice is random" was .67. Paper was not published in a journal.
2. In 1991 ...
3. In 2001 ...
4. In 2011 Bem ... the resulting p-value was .015. Paper was published.
5. In 2012 Richie, Wiseman, and French replicated 3x with p-values of .15, .40, and .38. Paper was rejected by the original journal and accepted by another journal.

1. Bem, D.J. (2011). Feeling the future: Experimental evidence for anomalous retroactive influences on cognition and affect. *Journal of Personality and Social Psychology, 100*, 407-425. doi: 10.1037/a0021524
2. Ritchie, S.J., Wiseman, R., and French, C.C. (2012). Failing the future: Three unsuccessful attempts to replicate Bem's 'retroactive facilitation of recall' effect. *Plos One, 7*. doi: 10.1371/journal.pone.0033423

How can the Replication Crisis be Addressed?

Open Science

1. Pre-registration and pre-registered reports
2. Multiple lab and multiple cohort studies
3. Replication studies executed by the authors or independent others
4. Publish data and analyses
5. Open access publications

As will be elaborated, (Bayesian) evaluation of informative hypotheses can contribute to Open Science.

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The Traditional Null Hypothesis

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

Cohen (1994) "The Earth is Round p <.05"

Royal (1997) "A power analysis should render $N = 0$"

Only use the null-hypothesis if it is a plausible representation of population of interest

1. Cohen, J. (1994). The earth is round, p<.05. *American Psychologist, 49,* 997-1003.
2. Royal, R. (1997). *Statistical Evidence. A Likelihood Paradigm.* New York: Chapman and Hall/CRC.

P-values and Alpha Level

p-value

The p-value is the probability of the observed data (or data that deviate more from H_0) assuming that H_0 is true.

The p-value is *not* a measure of support for the null-hypothesis, it is a measure of evidence *against* the null-hypothesis. It can therefore not be used to quantify the support in the data *for* the null-hypothesis.

P-values and Alpha Level

alpha level / Type I error

The probability of incorrectly rejecting the null-hypothesis.
The "usual" value is .05.

Were does the .05 come from? Fisher used "no level", .05, .02, .10, .01, and was in no way married to the .05.

Consequences of the .05 (or any other number): sloppy science, publication bias, ...

P-values and Alpha Level

Example

3 factors with corresponding group numbers:

	Condition		
	Masculine	Feminine	Neutral
Masculine Men	1	2	3
Feminine Men	4	5	6
Masculine Women	7	8	9
Feminine Women	10	11	12

Van Well, S., Kolk, A.M., Klugkist, I. (2008). Effects of Sex, Gender Role Identification, and Gender relevance of Two Types of Stressors on Cardiovascular and Subjective Responses: Sex and Gender Match/Mismatch Effects. Behavior Modification, 32, 427 - 449.

P-values and Alpha Level

Example: 3 × 2 × 2 ANOVA

Tests of Between-Subjects Effects

Dependent Variable: cs_sbp

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	17754.778[a]	12	1479.565	11.207	.000
Intercept	2145861.954	1	2145861.954	16253.783	.000
Baseline SBP	13049.137	1	13049.137	98.840	.000
Sekse	1339.880	1	1339.880	10.149	.002
GRI	76.680	1	76.680	.581	.448
Manipulation	180.911	2	90.456	.685	.507
Sekse*Manipulation	290.301	1	290.301	2.199	.142
Sekse*GRI	40.979	2	20.489	.155	.857
GRI*Manipulation	929.848	2	464.924	3.522	.034
Sekse*GRI*Manipulation	179.114	2	89.557	.678	.510
Error	10693.807	81	132.022		
Total	2280649.278	94			
Corrected Total	28448.586	93			

a. R Squared = .624 (Adjusted R Squared = .568)

P-values and Alpha Level

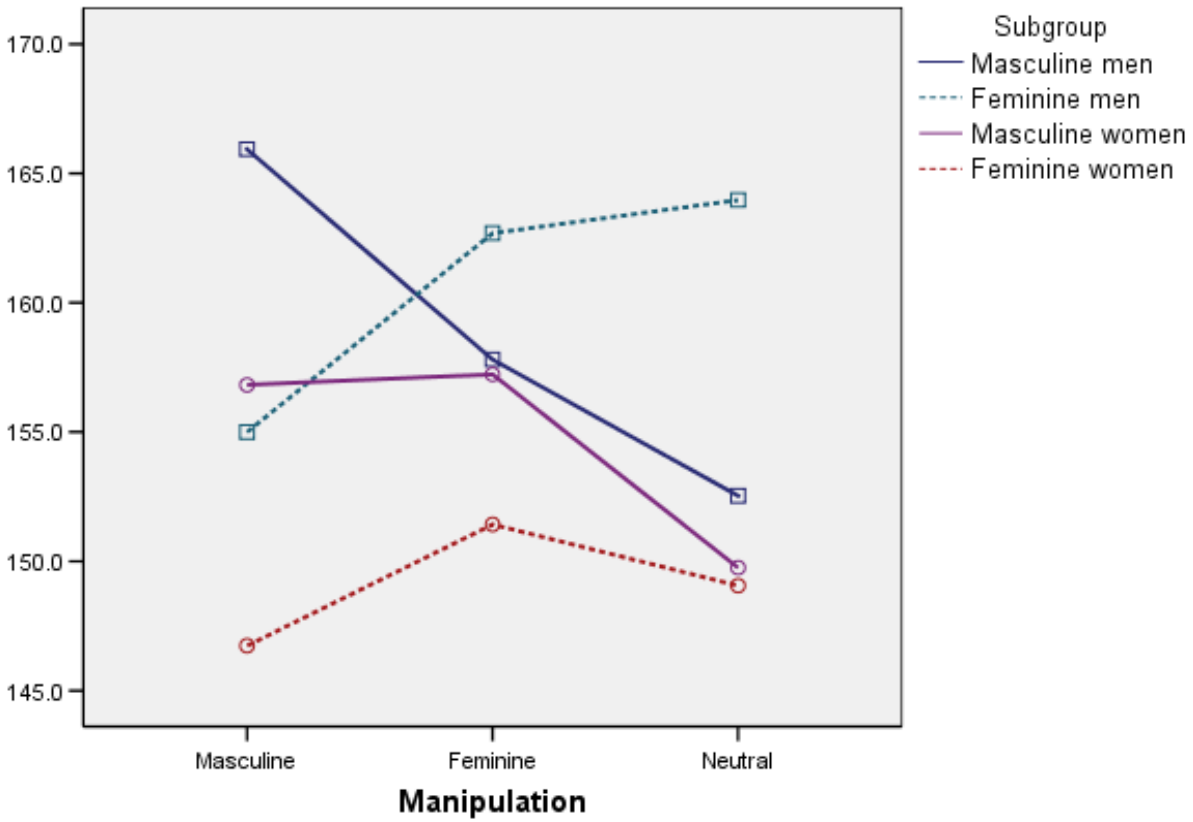
After observing ".06" (or .14 like on the previous slide) one *can not* update, that is, collect extra data and recompute the p-value. This procedure is called sequential data analysis. It has to be planned *before* the data is collected because it involves multiple evaluations of the hypotheses of interest and therefore the alpha level has to be corrected.

On top of that, how to deal with the fact that "the .05" is applied multiple times on the previous slide? How to correct for multiple hypotheses testing?

P-values and Alpha Level

Also, using an alpha level of .20 :-), we find four significant results.
It is clear that 'Something is going on, but we don't know what!'
And here we go eye-balling the data and effect sizes to interpret the results.

P-values and Alpha Level



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Informative Hypotheses

Example 1: ANOVA

	Condition		
	Masculine	Feminine	Neutral
Masculine Men	1	2	3
Feminine Men	4	5	6
Masculine Women	7	8	9
Feminine Women	10	11	12

Sex Match Effect

$$H_1 : (\mu_1, \mu_4) > (\mu_2, \mu_3, \mu_5, \mu_6) \text{ and } (\mu_8, \mu_{11}) > (\mu_7, \mu_9, \mu_{10}, \mu_{12})$$

Informative Hypotheses

ANOVA

	Condition		
	Masculine	Feminine	Neutral
Masculine Men	1	2	3
Feminine Men	4	5	6
Masculine Women	7	8	9
Feminine Women	10	11	12

Gender Role Match Effect

$$H_2 : (\mu_1, \mu_7) > (\mu_2, \mu_3, \mu_8, \mu_9) \text{ and } (\mu_5, \mu_{11}) > (\mu_4, \mu_6, \mu_{10}, \mu_{12})$$

Informative Hypotheses

ANOVA

	Condition		
	Masculine	Feminine	Neutral
Masculine Men	1	2	3
Feminine Men	4	5	6
Masculine Women	7	8	9
Feminine Women	10	11	12

Sex Mismatch Effect

$$H_3 : (\mu_2, \mu_5) > (\mu_1, \mu_3, \mu_4, \mu_6) \text{ and } (\mu_7, \mu_{10}) > (\mu_8, \mu_9, \mu_{11}, \mu_{12})$$

Informative Hypotheses

ANOVA

	Condition		
	Masculine	Feminine	Neutral
Masculine Men	1	2	3
Feminine Men	4	5	6
Masculine Women	7	8	9
Feminine Women	10	11	12

Gender Role Mismatch Effect

$$H_4 : (\mu_4, \mu_{10}) > (\mu_5, \mu_6, \mu_{11}, \mu_{12}) \text{ and } (\mu_2, \mu_8) > (\mu_1, \mu_3, \mu_7, \mu_9)$$

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bain

1. Hoijtink, H., Mulder, J., van Lissa, C., and Gu, X. (2018). A tutorial on testing hypotheses using the Bayes factor. *Psychological Methods, 24*, 539-556.

Bayes Factor

Balancing Fit and Complexity

The Bayes factor quantifies the relative support in the data for two hypotheses, for example,

$$H_i : \mu_1 > \mu_2 > \mu_3$$

$$H_u : \mu_1, \mu_2, \mu_3$$

with

$$BF_{iu} = \frac{f_i}{c_i} = \frac{\text{fit } H_i}{\text{complexity } H_i}$$

that is, after observing the data H_i is BF_{iu} times as likely as H_u, for example, .2, 5, 10.

Bayes Factor

Balancing Fit and Complexity

A (very) loose interpretation of the meaning of fit

$$H_i : \mu_1 > \mu_2 > \mu_3$$

if $\bar{x}_1 = 7$ & $\bar{x}_2 = 4$ & $\bar{x}_3 = 2$ the fit is good

if $\bar{x}_1 = 2$ & $\bar{x}_2 = 4$ & $\bar{x}_3 = 7$ the fit is bad

Bayes Factor

Balancing Fit and Complexity

A (very) loose interpretation of the meaning of complexity

$$H_1 : \mu_1 = \mu_2 = \mu_3$$

very parsimonious, the means have to be exactly equal.

$$H_1 : \mu_1 > \mu_2 > \mu_3$$

one ordering of three means: 1-2-3, thus is parsimonious.

$$H_2 : \mu_1 > (\mu_2, \mu_3)$$

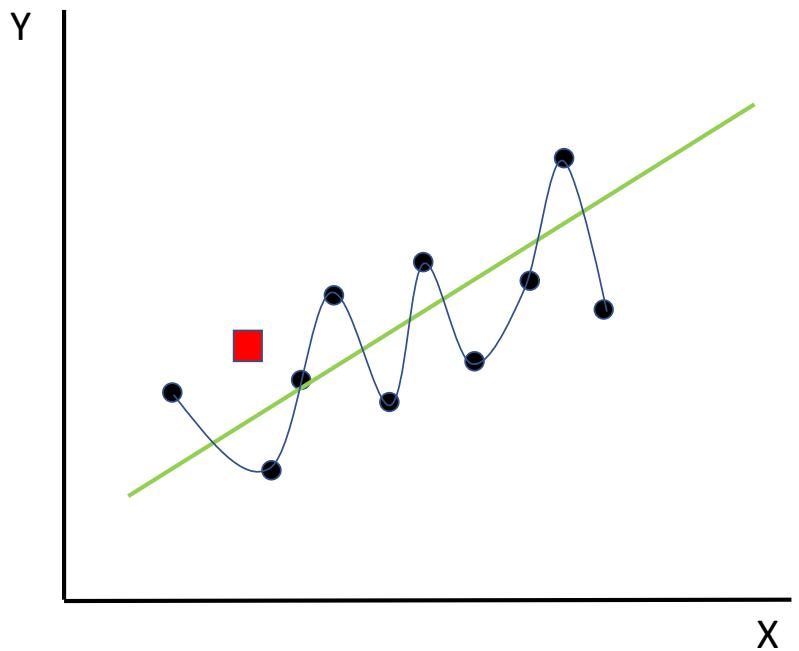
2 orderings of three means: 1-2-3 and 1-3-2, less parsimonious.

$$H_u : \mu_1, \mu_2, \mu_3$$

contains all six possible orderings of three means, not parsimonious.

Bayes Factor

Balancing Fit and Complexity



The straight line results from a linear regression model with 3 parameters (intercept, slope, residual variance).

The other line results from a polynomial regression models with 11 parameters (intercept, nine slopes, residual variance).

The red square is a new observation that is added to the original 10 observations.

What is the predictive value of both models?

Bayes Factor

Balancing Fit and Complexity

Three forms of Hypotheses and Bayes factors involving
$H_i : \mu_1 > \mu_2 > \mu_3$

BF_{iu} evaluating H_i versus $H_u : \mu_1, \mu_2, \mu_3$

$BF_{ii'}$ evaluating H_i versus $H_{i'} : \mu_1 = \mu_2 = \mu_3$

BF_{ic} evaluating H_i versus $H_c :$ not H_i

Bayes Factor

Interpreting (the Size of) the Bayes Factor

1. Select the best of a set of hypotheses using BF_{iu}
2. Compare two competing hypotheses using $BF_{ii'}$
3. Compare "my theory" with "not my theory" using BF_{ic}

	f_i	c_i	BF_{iu}	BF_{ic}
H_1 : Sex Match	.0039	.012	.32	.32
H_2 : Gender Role Match	.0725	.012	5.85	6.44
H_3 : Sex Mismatch	.0007	.012	.06	.06
H_4 : Gender Role Mismatch	.0001	.012	.01	.01

Bayes Factor

Descriptives

Gender Role Match Effect

$H_2 : (\mu_1, \mu_5) > (\mu_2, \mu_3, \mu_4, \mu_6)$ and $(\mu_7, \mu_{11}) > (\mu_8, \mu_9, \mu_{10}, \mu_{12})$

$H_2 : (166, 163) > (158, 154, 155, 164)$ and

$(157, 152) > (157, 150, 143, 149)$

Gender Role Mismatch Effect

$H_4 : (\mu_2, \mu_4) > (\mu_1, \mu_3, \mu_5, \mu_6)$ and $(\mu_8, \mu_{10}) > (\mu_7, \mu_9, \mu_{11}, \mu_{12})$

$H_4 : (158, 155) > (166, 154, 163, 164)$ and

$(157, 143) > (157, 150, 152, 149)$

Bayes Factor

Interpreting (the Size of) the Bayes Factor

1. The Bayes factor **is** a measure of support (also for the null-hypothesis)
2. The Bayes factor **can be indecisive**. A value around 1 denotes "the data don't tell us which hypothesis to prefer"
3. One **can update**, that is, collect more data and recompute the Bayes factor (see extra comments later on)
4. One **can compare** more than two hypotheses (see extra comments later on)
5. "Something is going on and **we do know what!**
6. The Bayes factor **selects the best of the hypotheses under consideration**. Note that the "true" hypothesis may not be among them, and that all hypotheses may be "wrong"

Bayes Factor

Interpreting (the Size of) the Bayes Factor

When is the Bayes factor large enough?

1. Guidelines by Jeffreys (1969) and Kass and Raftery (1995), e.g., < 3 is ignorable, > 3 is positive evidence, > 10 is strong evidence ...
2. Will lead to a return of sloppy science and publication bias (when used without pre-registration or a pre-registered report)
3. Were does the 3 come from?

Bayes Factor

Interpreting (the Size of) the Bayes Factor

When is the Bayes factor large enough?

1. Before collecting or accessing the data, formulate informative hypotheses (and decide how large you would like the Bayes factor to be).
2. Insert this information in a pre-registration or pre-registered report.
3. Collect data and evaluate hypotheses.
   - Is one good and the best with a "large" Bayes factor: nice!
   - Are the Bayes factors "not large enough": follow up research or updating is needed.
   - Is none good: BIG news, well-constructed hypotheses have been rejected!

Bayesian Error Probabilities

Posterior Model Probabilities, e.g., $PMP(H_i \mid \text{data})$ and $PMP(H_c \mid \text{data})$ quantify the support in the data for each hypothesis.

$$\frac{PMP(H_i|\text{data})}{PMP(H_c|\text{data})} = \text{BF}_{ic} \times \frac{PRI(H_i)}{PRI(H_c)}, \tag{1}$$

where $PRI(H_i)$ and $PRI(H_c)$ denote the *prior* probabilities, that is, an evaluation of the support for the hypotheses *before* observing the data.

Usually equal prior model probabilities are used (which means that the PMP's convey the same information as the Bayes factors), but this is not a requirement.

PMPs

PMPs can be interpreted as Bayesian error probabilities, that is, the Bayesian counterparts of the Type I and Type II errors.

	f_i	c_i	BF_{iu}	PMP_i	PRI_i
H_1 : Sex Match	.0039	.012	.32	.04	1/5
H_2 : Gender Role Match	.0725	.012	5.85	.81	1/5
H_3 : Sex Mismatch	.0007	.012	.06	.01	1/5
H_4 : Gender Role Mismatch	.0001	.012	.01	.00	1/5
H_u :				.14	1/5

PMPs

H_i contains 1 ordering of means:

1. $\mu_1 > \mu_2 > \mu_3$

H_c contains 5 orderings of means:

2. $\mu_1 > \mu_3 > \mu_2$
3. $\mu_2 > \mu_1 > \mu_3$
4. $\mu_2 > \mu_3 > \mu_1$
5. $\mu_3 > \mu_1 > \mu_2$
6. $\mu_3 > \mu_2 > \mu_1$

H_u combines H_i and H_c.

PMPs

Replacing H_u by H_c

	f_i	c_i	BF_{iu}	PMP_i	PRI_i
H_1 : Sex Match	.0039	.012	.32	.04	1/5
H_2 : Gender Role Match	.0725	.012	5.85	.84	1/5
H_3 : Sex Mismatch	.0007	.012	.06	.00	1/5
H_4 : Gender Role Mismatch	.0001	.012	.01	.00	1/5
H_c :	.9200	.9500	.97	.12	1/5

Where H_c denotes the complement H_1 through H_4, that is, "not one of these four hypotheses".

PMPs

The Number of Hypotheses and PMPs

Look what happens if we compare many hypotheses, the PMPs become smaller and smaller, and thus the Bayesian error probabilities become larger and larger:

	f_i	c_i	BF_{iu}	PMP_i	PRI_i
H_1 : Sex Match	.0039	.012	.32	.013	1/13
H_2 : Gender Role Match	.0725	.012	5.85	.270	1/13
H_3 : Sex Mismatch	.0007	.012	.06	.003	1/13
H_4 : Gender Role Mismatch	.0001	.012	.01	.000	1/13
H_5 : Lets try this one too	.0521	.012	2.61	.180	1/13
...					
H_{12} : Don't miss something	.0164	.012	1.36	.040	1/13
H_u :				.047	1/13

PMPs

The Number of Hypotheses and PMPs

The same results as two slides up are in fact obtained by assigning PMPs of 0 to each hypothesis that is NOT considered:

	f_i	c_i	BF_{iu}	PMP_i	PRI_i
H_1 : Sex Match	.0039	.012	.32	.04	1/5
H_2 : Gender Role Match	.0725	.012	5.85	.81	1/5
H_3 : Sex Mismatch	.0007	.012	.06	.01	1/5
H_4 : Gender Role Mismatch	.0001	.012	.01	.00	1/5
H_5 : Lets try this one too	.0521	.012	2.61	.18	0
...					
H_{12} : Don't miss something	.0164	.012	1.36	.04	0
H_u :				.14	1/5

Subjectivity of Bayesian Hypotheses Evaluation

1. Which hypotheses to evaluate?
2. How to formalize hypotheses?
   E.g. $(\mu_1, \mu_2) > (\mu_3, \mu_4)$ or $\mu_1 = \mu_2 > \mu_3 = \mu_4$
3. The (implicit) choice for equal prior model probabilities
4. The specification of the prior distribution

Bayesian Updating

1. Rouder, J. N. (2014). Optional stopping: No problem for Bayesians. *Psychonomic Bulletin & Review, 21,* 301-308.

Bayesian Updating

Repeated significance testing after increasing the sample requires "planning before the data collection has started" and "correction for multiple testing".

"Bayesian updating" is simply recomputing the evidence presented by all the data that are currently available. This can both be done using the Bayes factor "what is the relative support in the available data for this pair of hypotheses" and/or the PMPs "what is the support in the available data for this hypothesis".

Bayesian Updating

Bayes factors	*N* per group			
	8	+8	+12	+5
H_1 : Sex Match	.32	.24	.12	.02
H_2 : Gender Role Match	5.85	7.12	9.23	11.82
H_3 : Sex Mismatch	.06	.02	.00	.00
H_4 : Gender Role Mismatch	.01	.00	.00	.00

PMPs	*N* per group			
	8	+8	+12	+5
H_1 : Sex Match	.04	.03	.01	.00
H_2 : Gender Role Match	.84	.86	.91	.93
H_3 : Sex Mismatch	.00	.00	.00	.00
H_4 : Gender Role Mismatch	.00	.00	.00	.00
H_c	.12	.11	.08	.07

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Informative Hypotheses

Example 1: ANOVA

What is the relation between "knowledge of numbers after watching Sesame Street for a year"

and

site from which the child originates (1 = disadvantaged inner city, 2 = advantaged suburban , 3 = advantaged rural, 4 = disadvantaged rural, 5 = disadvantaged Spanish speaking).

Informative Hypotheses

Example 1: ANOVA

```
library(bain)
sesamesim$site <- as.factor(sesamesim$site)
anov <- lm(postnumb~site-1,sesamesim)
coef(anov)
set.seed(100)
results <- bain(anov,
                "site1=site2=site3=site4=site5;
                 site2>site5>site1>site3>site4")
print(results)
summary(results, ci = 0.95)
```

Informative Hypotheses

Example 1: ANOVA

coef(anov) renders

```
   site1     site2     site3     site4     site5
29.66667 38.98182 23.18750 25.32558 31.72222
```

summary(results) renders

```
  Parameter  n Estimate       lb       ub
1     site1 60 29.66667 26.82991 32.50343
2     site2 55 38.98182 36.01892 41.94472
3     site3 64 23.18750 20.44082 25.93418
4     site4 43 25.32558 21.97466 28.67650
5     site5 18 31.72222 26.54303 36.90141
```

Informative Hypotheses

Example 1: ANOVA

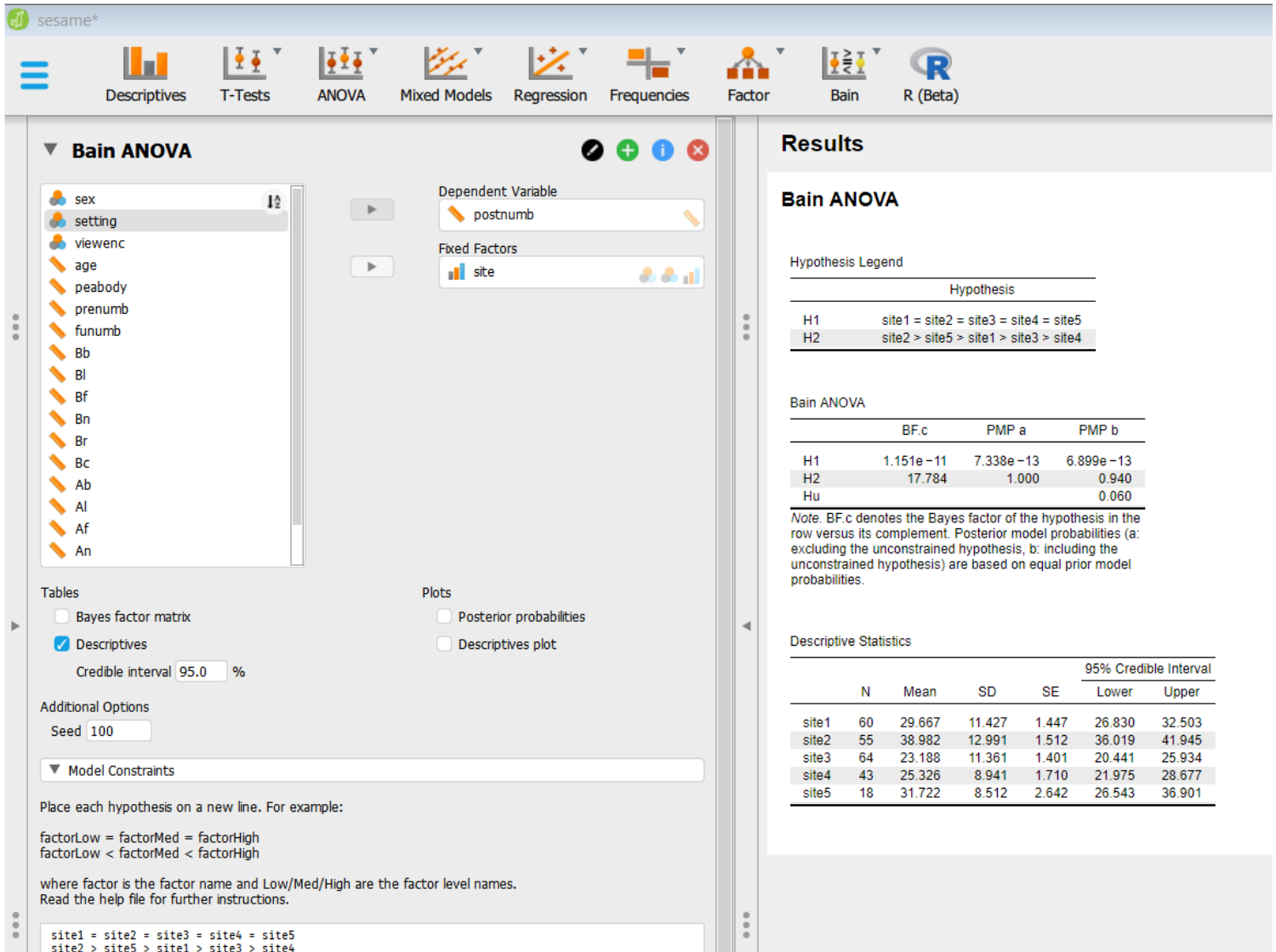
The main output is

```
    Fit    Com    BF.u    BF.c    PMPa   PMPb   PMPc
H1 0.000 0.000 0.000   0.000   0.000 0.000 0.000
H2 0.121 0.008 14.559 16.428 1.000 0.936 0.943
Hu                                    0.064
Hc 0.879 0.992 0.886                        0.057

Hypotheses:
  H1: site1=site2=site3=site4=site5
  H2: site2>site5>site1>site3>site4
```

Informative Hypotheses

Example 1: ANOVA



Informative Hypotheses

Example 2: ANOVA Interaction Effect

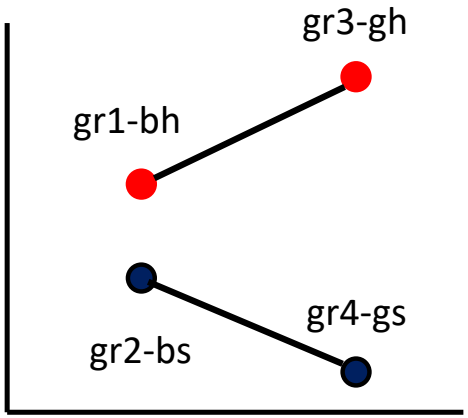
Dependent variable: Knowledge of numbers.

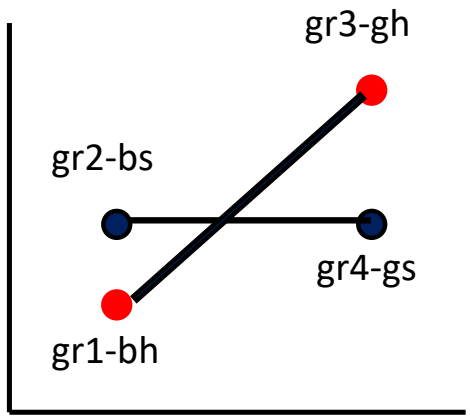
Factors: sex (boy, girl) and setting (watching at home, watching at school).

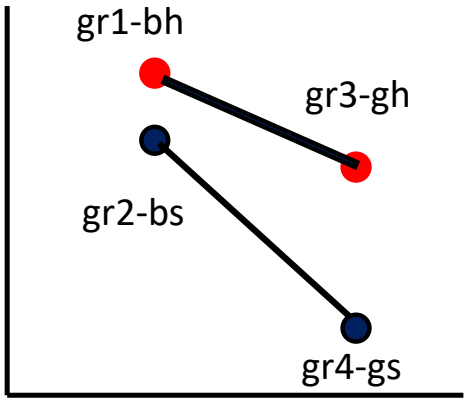
Gr: 1=boyhome, 2= boyschool, 3= girlhome, 4=girlschool.

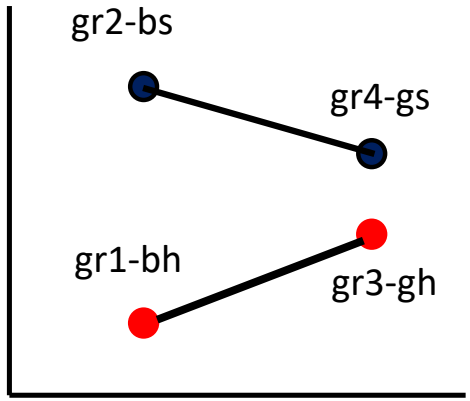
Informative Hypotheses

Example 2: ANOVA Interaction Effect









$$Hi:$$

$$gr2-gr1 > gr4-gr3$$

and

$$gr2 > gr1$$

$$gr2 > gr4$$

Informative Hypotheses

Example 2: ANOVA Interaction Effect

```
sesamesim$gr <- as.factor(sesamesim$gr)
anov <- lm(postnumb~gr-1,sesamesim)
results <- bain(anov,
"gr2 - gr1 > gr4 - gr3 & gr2 > gr1 & gr2 > gr4")
```

Informative Hypotheses

Example 2: ANOVA Interaction Effect

The main output is

```
    Fit   Com   BF.u  BF.c   PMPa  PMPb  PMPc
H1 0.922 0.283 3.262 29.984 1.000 0.765 0.968
Hu                                 0.235
Hc 0.078 0.717 0.109                     0.032

Hypotheses:
  H1: gr2-gr1>gr4-gr3&gr2>gr1&gr2>gr4
```

Informative Hypotheses

How to write down an hypothesis

bain can handle hypotheses build using constraints on (linear combinations) of parameters. Suppose the parameter names are "a", "b", "c".

Step 1: Construct the elements of the linear combination. E.g. "a" or "a + 2" or "3 * a" or "2 * a + 4"

Step 2: Constrain the resultsing elements. E.g. a > b > c

or a > b + 2 & b > c + 2

or 2 * a > b + c & b > 0 & c > 0

or a > (b, c) & b - c > 0

Informative Hypotheses

Example 3: Repeated Measures

Development of depression

	Measurement			
	8 years	12 years	16 years	20 years
Men	μ_1	μ_2	μ_3	μ_4
Women	μ_5	μ_6	μ_7	μ_8

$$H_1 : \mu_5 - \mu_1 > \mu_6 - \mu_2 > \mu_7 - \mu_3 < \mu_8 - \mu_4$$

$$H_2 : \mu_6 - \mu_5 < \mu_7 - \mu_6 > \mu_8 - \mu_7$$

Informative Hypotheses

Example 4: Multiple Regression

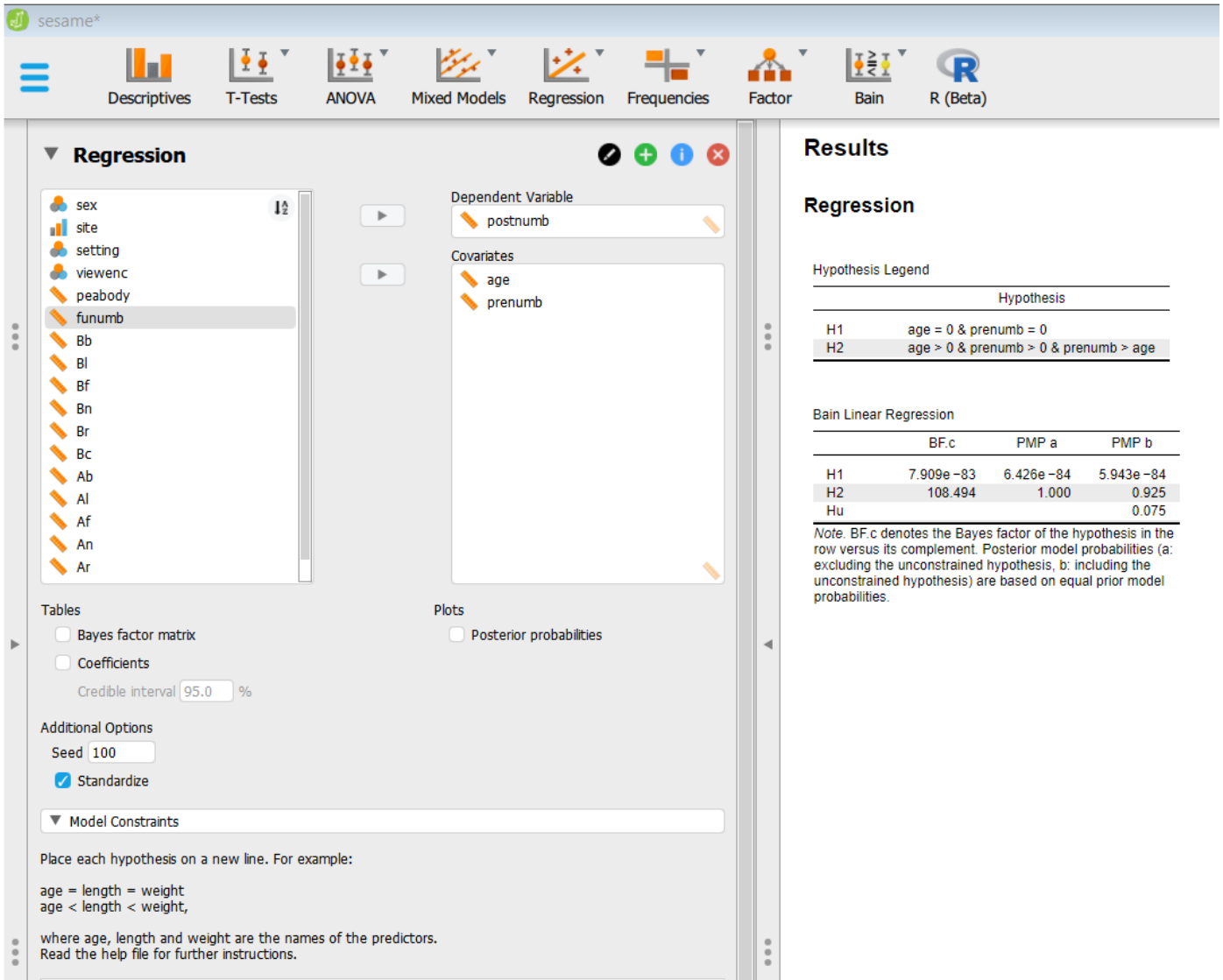
$$\text{postnumb}_i = \beta_0 + \beta_1 \times \text{age}_i + \beta_2 \times \text{prenumb}_i + \epsilon_i$$

$$H_1 : \beta_1 > 0, \beta_2 > 0, \beta_1 < \beta_2$$

Note: β_1 and β_2 are only comparable if age and prenumb are standardized

Informative Hypotheses

Example 4: Multiple Regression



Informative Hypotheses

Example 5: About Equality Constraints

Is the difference in number knowledge relevantly different between boys and girls?

Informative Hypotheses

Example 5: About Equality Constraints

```
sesamesim$sex <- as.factor(sesamesim$sex)
anov <- lm(postnumb~sex-1,sesamesim)
results <- bain(anov, "-2 < sex1 - sex2 < 2")
```

Informative Hypotheses

Example 5: About Equality Constraints

```
sex1      sex2
30.09565 28.85600

   Fit   Com   BF.u  BF.c   PMPa  PMPb  PMPc
H1 0.664 0.091 7.304 19.735 1.000 0.880 0.952
Hu                                 0.120
Hc 0.336 0.909 0.370                     0.048

Hypotheses:
  H1: -2<sex1-sex2<2
```

Informative Hypotheses

Example 6: Structural Equation Modelling

```
library(bain)
library(lavaan)

model <- '
    A  =~ Ab + Al + Af + An + Ar + Ac
    B  =~ Bb + Bl + Bf + Bn + Br + Bc
    A  ~ B + age + peabody'
fit <- sem(model, data = sesamesim, std.lv = TRUE)

hypotheses <- "A~B = A~peabody = A~age = 0;
               A~B > A~peabody > A~age = 0"
set.seed(100)
y1 <- bain(fit, hypotheses, standardize = TRUE)
```

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Null Hypothesis Significance Testing (NHST)

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ANOVA and Beyond, Example Analyses with R and JASP

Extra

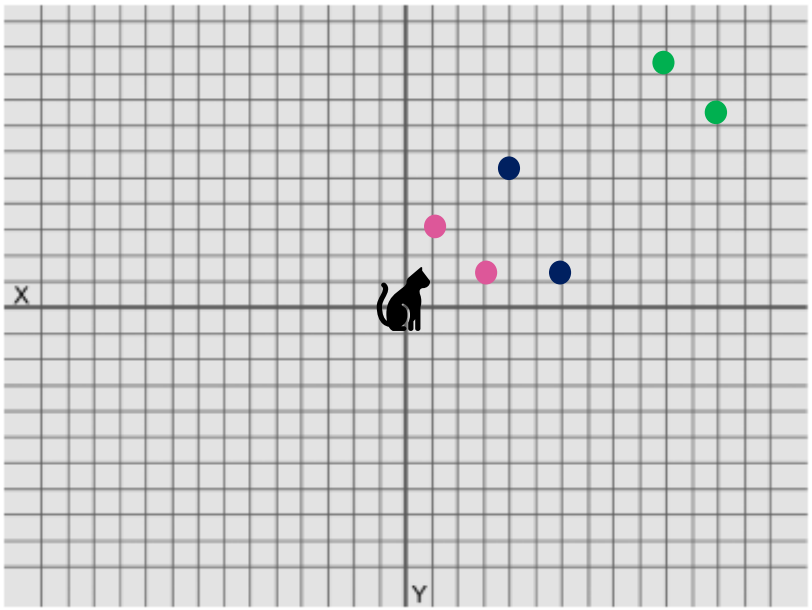
bain Evaluation of Replication Studies

1. Leplaa, H., Rietbergen, C., and Hoijtink, H. (unpublished). Bayesian Evaluation of Replication Studies.

A Research Project and Its Replication

An experiment with three conditions:

- The "close" condition
- The "intermediate" condition
- The "distant" condition



Participants Rated:

Attachment to:

- Siblings
- Parents
- Home-town

on a

1 (not at all strong) – 7 (extremely strong)
Likert scale

which are
averaged to obtain the dependent variable

The description given here is a modification of and inspired by the actual experiment executed by Williams, L.E. and Bargh, J.A. (2008). Keeping One's Distance. The Influence of Spatial Distance Cues on Affect and Evaluation. *Psychological Science, 19,* 302-308.

The Main Research Outcomes

Williams and Bargh (2008) tested:

H_0: $\mu_{close} = \mu_{intermediate} = \mu_{distant}$,

that is, the three means are equal

rendering

p-value = .01, that is, smaller than .05, that is, the means are significantly different

with

$m_{close} = 5.61$, $m_{intermediate} = 5.23$, $m_{distant} = 4.86$

and

$\eta^2 = .11$, that is, the three conditions explain 11% of the variation in attachment, which is a medium to strong effect of condition

The replication by Joy-Gaba, Clay, and Cleary (2016) rendered

p-value = .79

with

$m_{close} = 5.44$, $m_{intermediate} = 5.31$, $m_{distant} = 5.31$

And

$\eta^2 = .00$

Joy-Gaba, J., Clay, R., and Cleary, H. (2016). Replication of keeping one's distance: The influence of spatial distance cues on affect and evaluation by Williams L.E. and Bargh J.A. (2008) *Psychological Science, 19,* 302-308). Retrieved from https://osf.io/a78bm/

Replication Research

Replication Hypotheses Derived from the Original Study

$$\mu_{close} > \mu_{intermediate} > \mu_{distant}$$

$$\mu_{close} > \mu_{intermediate} + .2sd \;\&\; \mu_{intermediate} > \mu_{distant} + .2sd$$

Replication Research

Replication Hypotheses Derived from the Original Study

```
    Fit   Com   BF.u  BF.c  PMPa  PMPb  PMPc
H1 0.298 0.168 1.780 2.112 1.000 0.640 0.680
Hu                               0.360
Hc 0.702 0.832 0.843                   0.320


Hypotheses:
  H1: Close>Interm>Dist
```

Replication Research

Replication Hypotheses Derived from the Original Study

```
    Fit   Com   BF.u  BF.c  PMPa  PMPb  PMPc
H1 0.025 0.168 0.147 0.125 1.000 0.128 0.111
Hu                               0.872
Hc 0.975 0.832 1.171                 0.899


Hypotheses:
  H1:Close>Interm+.20&Interm>Dist+.20
```

A Closer Look at the Bayes Factor

A Closer Look at the Bayes Factor

Three Simple Hypotheses

Consider the hypotheses:

$$H_1 : \mu_1 \approx \mu_2, \text{ that is, } |\mu_1 - \mu_2| < .1$$

$$H_2 : \mu_1 > \mu_2$$

$$H_3 : \mu_1, \mu_2$$

A Closer Look at the Bayes Factor

Information in the Data about the Two Means

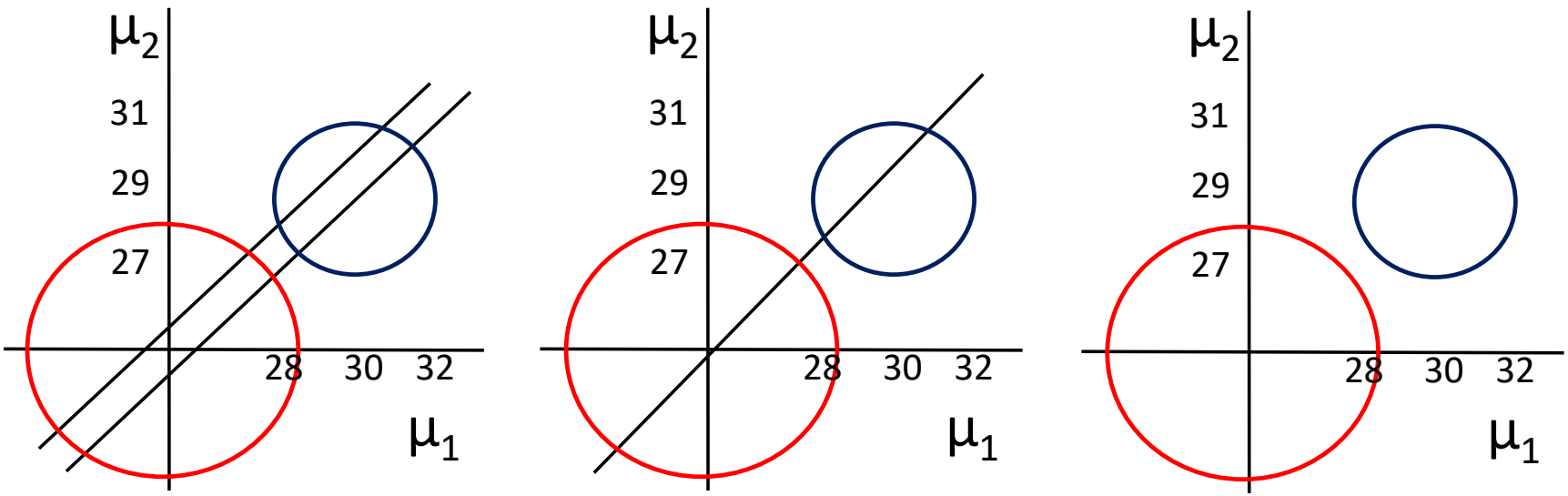
	N	Mean	SD	SE	95% Credible Interval Lower	95% Credible Interval Upper
sex1	115	30.096	13.058	1.175	27.793	32.398
sex2	125	28.856	12.162	1.127	26.647	31.065

$$g(\mu_1, \mu_2 \mid \text{data}) \approx \mathcal{N}\left(\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \begin{bmatrix} se_1^2 = \frac{SD_1^2}{N_1} & 0 \\ 0 & se_2^2 = \frac{SD_2^2}{N_2} \end{bmatrix} \right),$$

A Closer Look at the Bayes Factor

Posterior Distribution, Prior Distribution, and Hypotheses

$H_1: \mu_1 \approx \mu_2$ $H_2: \mu_1 > \mu_2$ $H_u: \mu_1 , \mu_2$



$BF_{1u} = f_1/c_1 = .25/.05 = 5$ $BF_{2u} = f_2/c_2 = .75/.5 = 1.5$

$BF_{12} = 5/1.5 = 3.33$

A Closer Look at the Bayes Factor

Fit and Complexity

1. The fit of a hypothesis is the proportion of the posterior distribution in agreement with the hypothesis.
2. The complexity of a hypothesis is the proportion of the prior distribution in agreement with the hypothesis.

A Closer Look at the Bayes Factor

The Prior Distribution

$$h(\mu_1, \mu_2 \mid \text{data}) \approx \mathcal{N}\left(\begin{bmatrix} m \\ m \end{bmatrix}, \begin{bmatrix} \frac{SD_1^2}{J} & 0 \\ 0 & \frac{SD_2^2}{J} \end{bmatrix} \right),$$

Where μ_1 and μ_2 have the same prior mean m, and where J denotes the size of the training sample.

Choices for J for the example at hand:

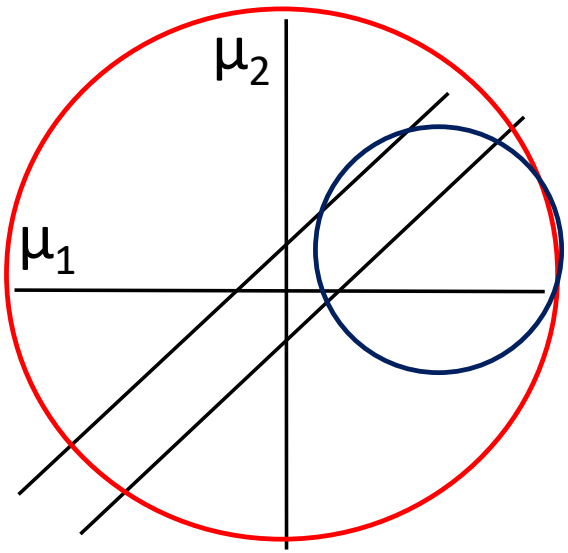
- Default in `bain`: number of independent constraints, that is, 1, this is a conservative choice (NB. $.5 * J$)
- Minimal training sample size, that is, 4, because four observations are neede to estimate two means and variances
- *Jref*, which renders $BF_{0u} = 19$ if the effect size in the sample equals 0

A Closer Look at the Bayes Factor

Prior Sensitivity for Constrained Hypotheses

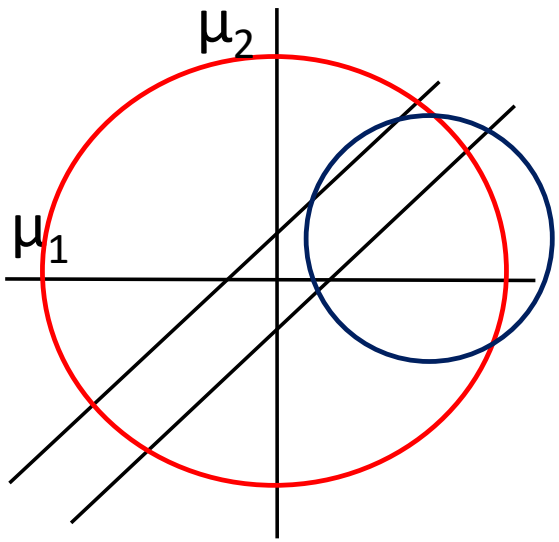
$H_1: \mu_1 \approx \mu_2$

J = 1



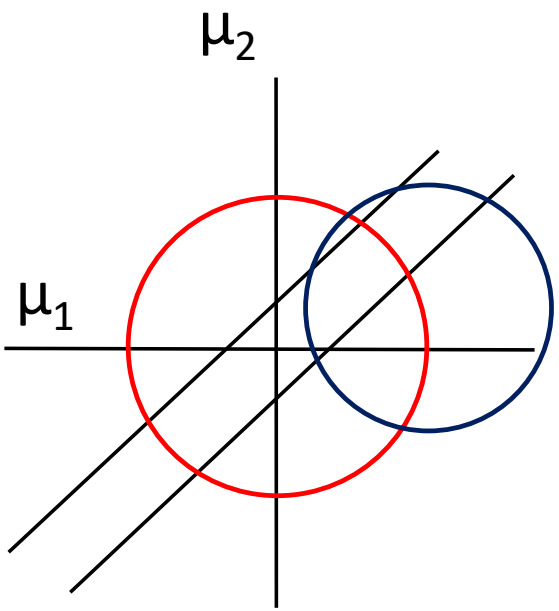
$BF_{1u} = .2/.01 = 20$

J =2



$BF_{1u} = .2/.05 = 4$

J = 3



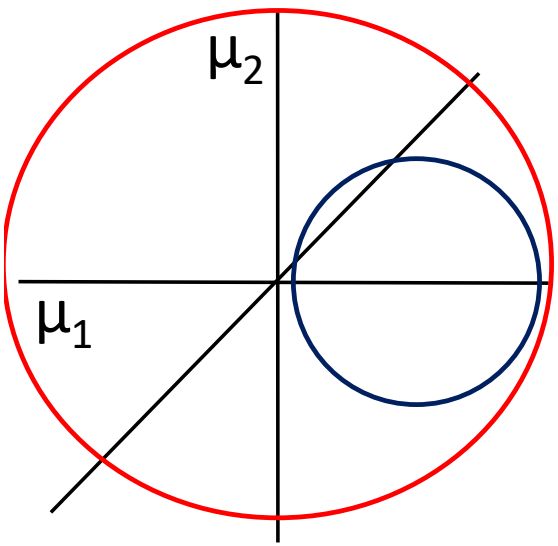
$BF_{1u} = .2/.2 = 1$

A Closer Look at the Bayes Factor

Prior In-Sensitivity for > , < Constrained Hypotheses

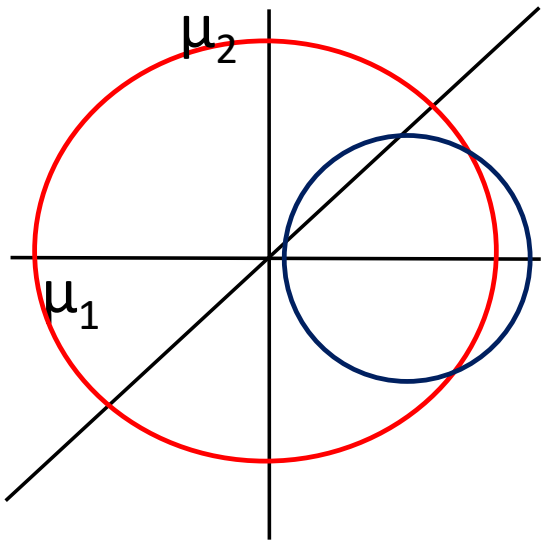
$H_2: \mu_1 > \mu_2$

J=1



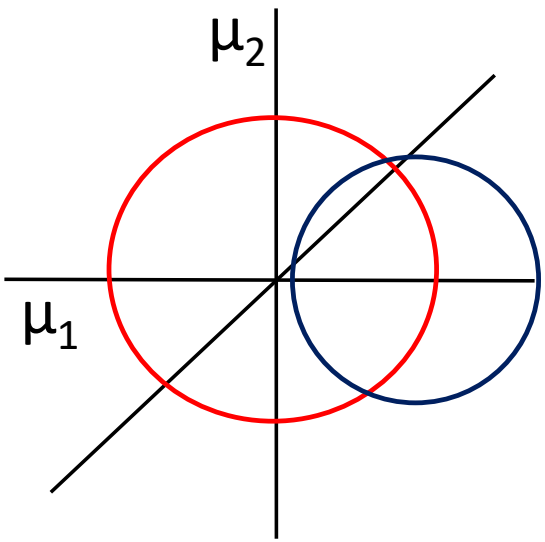
$BF_{2u} = .9/.5 = 1.8$

J=2



$BF_{2u} = .9/.5 = 1.8$

J=3



$BF_{2u} = .9/.5 = 1.8$

Bayes Factor (BF)

comparing two informative hypotheses

The BF quantifies the relative support in the data for two hypotheses.

$$BF_{12} = \frac{BF_{1u}}{BF_{2u}} = \frac{f_1/f_2}{c_1/c_2}$$

using

$$BF_{iu} = \frac{f_i/f_u}{c_i/c_u} = \frac{f_i}{c_i}$$