```
11 per  ~ sat   0.151 0.105 1.440  0.150
12 per  ~ tra   0.286 0.084 3.403  0.001
⋮
```

Note that the label `per ~ kno` denotes the coefficient θ_1 which relates η to ξ_1 in the regression model (46). We only show the results for the four regression coefficients used in (47), (48), and (49). The standardized estimates of the target parameters are given in the column under `est.std`. For example, the estimate of θ_1 is 0.478 in the row of `per ~ kno`, and the estimate of θ_4 is 0.286 in the row of `per ~ tra`.

The output of `ZVCOV[9:12,9:12]` renders the standardized covariance matrix of $\theta_1, \ldots, \theta_4$:

```
             per~kno       per~ori       per~sat       per~tra
per~kno  0.026034895 -0.0223249106 -0.0050273595 -0.0011610045
per~ori -0.022324911  0.0273346337  0.0043904540 -0.0007619234
per~sat -0.005027359  0.0043904540  0.0110250662 -0.0002713825
per~tra -0.001161004 -0.0007619234 -0.0002713825  0.0070519650
```

The standardized estimates and covariance matrix of target parameters obtained in `lavaan` can be used as input for `BF-SEM`. This will be shown in the user manual in Appendix B.

Appendix B. User Manual of BF-SEM

`BF-SEM` is a `Fortran 90` program developed in Microsoft Visual Studio 2005 with the IMSL 5.0 `Fortran` numerical library. This software package is free and available at http://informative-hypotheses.sites.uu.nl/software/. The downloadable folder contains an executable file `BF-SEM`.exe, and text files Input.txt and Output.txt for the two examples used in this paper. This section provides the user manual of `BF-SEM` such that researchers can use it for the evaluation of order constrained hypotheses by means of Bayes factors. The input for `BF-SEM` contains the estimates and covariance matrix of target parameters obtained in `lavaan` and the restriction matrix $[\boldsymbol{R}_i|\boldsymbol{r}_i]$ for each hypothesis under consideration. With this input, running `BF-SEM` renders the Bayes factor and PMP for each hypothesis. We will use the example from Section 6.2 to illustrate the use of `BF-SEM`.

Appendix B.1. Input file

The Input.txt and `BF-SEM`.exe files have to be located in the same folder. The input file, e.g., for the regression model from Section 6.2 can be found

below:

```
1   #Numbers of target parameters and hypotheses under consideration
2   4 3
3   #Estimates of parameters
4   0.784 0.550 0.248 0.471
5   #Covariance matrix of parameters
6    0.026034895 -0.0223249106 -0.0050273595 -0.0011610045
7   -0.022324911  0.0273346337  0.0043904540 -0.0007619234
8   -0.005027359  0.0043904540  0.0110250662 -0.0002713825
9   -0.001161004 -0.0007619234 -0.0002713825  0.0070519650
10  #Number of constraints in hypothesis 1
11  3
12  #Restriction matrix (R|r) for hypothesis 1
13  1 -1  0  0   0
14  0  1  0 -1   0
15  0  0 -1  1   0
16  #Number of constraints in hypothesis 2
17  3
18  #Restriction matrix (R|r) for hypothesis 2
19  1 -1  0  0   0
20  0  1 -1  0   0
21  0  0  1 -1   0
22  #Number of constraints in hypothesis 3
23  3
24  #Restriction matrix (R|r) for hypothesis 3
25  -1  1  0  0   0
26   0 -1  1  0   0
27   0  0 -1  1   0
```

Note that the structure of the input file cannot be changed. Both the lines containing annotation starting with `#` and the lines with numbers have to be presented. As can be seen on the second line, there are 4 target parameters in the regression model and 3 competing hypotheses with respect to those parameters. On the fourth line, the estimates of parameters obtained from `lavaan` are given, and on line six through nine the covariance matrix is given. The eleventh line shows that hypothesis 1 can be specified using 3 constraints. Next, there are three lines under the label `#Restriction matrix (R|r) for hypothesis 1`, each of which expresses a constraint in hypothesis 1. This will be elaborated in detail in the next paragraph. Because the second line shows that 3 hypotheses have to be evaluated, we need to specify two extra hypotheses (hypothesis 2 and hypothesis 3) for which the numbers of

constraints and the restriction matrices can be placed in a similar fashion as for hypothesis 1.

As was shown in Section 2.2, an order constrained hypothesis H_i can be formulated by $\boldsymbol{R}_i\theta > \boldsymbol{r}_i$. Each constraint $\boldsymbol{R}_{ik}\boldsymbol{\theta} > r_{ik}$ for $k = 1, \ldots, K$ in the hypothesis can be written as $R_{ik1}\theta_1 + \ldots + R_{ikJ}\theta_J > r_{ik}$, where K and J are numbers of constraints and parameters in H_i, respectively. Note that every parameter should be moved to the left hand side of the inequality sign ">", and the constant should be moved to the right hand. In the restriction matrix (R|r), the constraint $\boldsymbol{R}_{ik}\boldsymbol{\theta} > r_{ik}$ can be expressed by the line

$R_{ik1}\ R_{ik2}\ \ldots\ R_{ikJ} \quad r_{ik}$.

For example,

- $\theta_1 + \theta_2 + \theta_3 > 0$ corresponds to
  1 1 1 0
- $\theta_1 - 2\theta_2 + 3\theta_3 > 0.5$ corresponds to
  1 -2 3 0.5
- $\theta_1 - 2 > \theta_2 - \theta_3$ corresponds to
  1 -1 1 2
- $\theta_1 > \theta_2 > \theta_3$ corresponds to
  1 -1 0 0
  0 1 -1 0
- $\theta_1 - \theta_2 > \theta_3 - \theta_4 > \theta_5 - \theta_6$ corresponds to
  1 -1 -1 1 0 0 0
  0 0 1 -1 -1 1 0

Thus, below the label `#Restriction matrix (R|r) for hypothesis 1`, the three lines

```
1 -1  0  0   0
0  1  0 -1   0
0  0 -1  1   0
```

represent the hypothesis $\theta_1 > \theta_2 > \theta_4 > \theta_1$ in the regression model.

It should be noted that the equality, about equality, and range constrained hypotheses can not be evaluated using `BF-SEM`. Therefore, the restriction matrix (R|r)

```
 1 -1   0
-1  1   0
```

is not allowed, because it implies an equality constrained hypothesis $\theta_1 = \theta_2$. The restriction matrix (R|r)

$$\begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \quad \begin{array}{c} -d \\ d \end{array}$$

is not allowed, because it implies an about equality constrained hypothesis $|\theta_1 - \theta_2| < d$, where d represents the tolerable deviation. The restriction matrix (R|r)

$$\begin{array}{ccc} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{array}$$

is not allowed, because it implies a range constrained hypothesis $0 < \theta_1 < \theta_2 < 1$. The restriction matrix (R|r)

$$\begin{array}{cc} 1 & 1 \\ -1 & 0 \end{array}$$

is not allowed, because it implies $\theta_1 > 1$ and $\theta_1 < 0$ which contradict each other. If the restriction matrix (R|r) contains any equality, about equality, range, or contradicting constraints, executing `BF-SEM` will produce an error message:

```
WARNING: Hypothesis i contains equality, about equality, range, or
contradicting constraints!
```

Besides the input of inappropriate hypotheses, there are four possible ways of making errors in the Input.txt file. First, one may by accident delete the annotate line starting with `#`. This results in the error message:

```
WARNING: Miss an annotate line in Input.txt!
```

Second, the length of the estimates and the rank of the covariance matrix of parameters are not in line with the number of target parameters specified in the second line. This results in the error message:

```
WARNING: An error below "#Estimates of parameters" in Input.txt!
```

or

```
WARNING: An error below "#Covariance matrix of parameters" in Input.txt!
```

Third, the number of lines below `#Restriction matrix (R|r)` is not in line with the number below `#Number of constraints in hypothesis i`. This results in the error message:

```
WARNING: An error below #Restriction matrix (R|r) for hypothesis
i in Input.txt!
```

Fourth, when two or more hypotheses are under consideration, one may forget to specify the number of constraints and the restriction matrix (R|r) for every hypothesis. This results in the error message:

```
Hypothesis i needs to be specified in Input.txt!
```

If an unknown problem occurs when running `BF-SEM`.exe, please send your Input.txt to `x.gu@uu.nl`.

Appendix B.2. Output file

Executing `BF-SEM`.exe renders a text file Output.txt in the same folder. If there already exists an Output.txt, it will be overwritten by the new one. Output.txt not only displays Bayes factors and PMPs for order constrained hypotheses, but also the decomposed fits and complexities with the corresponding numbers of iterations in Gibbs sampler. It should be noted that the output of `BF-SEM` also displays the complexities, Bayes factors, and PMPs under prior with diagonal covariance structure presented by Gu et al. (2014) for comparison. This prior is referred to as prior 1. In contrast, the prior (16) used in this paper is referred to as prior 2. Applied researchers are advised only using prior 2, as it holds invariance property for linear transformation. The output file corresponding to Input.txt shown in the previous section is:

```
Result for hypothesis  1
     Fits                numbers of iterations
    0.6881                     4000
    0.4330                     4000
    0.7631                     4000
Complexities (prior 1)   numbers of iterations
    0.5015                     4000
    0.3379                     4000
    0.2514                     4000
Complexities (prior 2)   numbers of iterations
    0.4970                     4000
    0.1908                     4000
    0.2534                     4000
Total fit      Complexity (prior1)     Complexity (prior 2)
  0.2274             0.0426                 0.0240
BFiu (prior 1)    BFiu (prior 2)
  5.3367             9.4612

BFic (prior 1)    BFic (prior 2)
  6.6129             11.9512

Result for hypothesis  2
⋮

Result for hypothesis  3
⋮
```

```
Result of PMP for each hypothesis
PMP (prior1)      PMP (prior2)    for hypothesis  1
  0.8041             0.7840
PMP (prior1)      PMP (prior2)    for hypothesis  2
  0.1950             0.2152
PMP (prior1)      PMP (prior2)    for hypothesis  3
  0.0009             0.0009
```

The output file contains the Bayes factors and PMPs for each hypothesis under consideration. The interpretations of Bayes factors and PMPs are elaborated in Section 3 in this paper. As shown in Section 3, the Bayes factor can be computed by multiplying the decomposed fits divided by decomposed complexities. For the result of hypothesis 1, first of all three decomposed fits are displayed below the label `Fits`, and the corresponding numbers of iterations used for the computation of the fits are shown on the right side. Then the decomposed complexities under prior 1 are presented below the label `Complexities (prior 1)`, which is followed by the complexities under prior 2. The numbers of iterations used to obtain these complexities are placed in the corresponding line. Thereafter, the fit, and complexities under prior 1 and 2 for hypothesis 1 can be obtained by multiplying the decomposed fits and complexities, which are shown under the labels `Total fit`, `Complexity (prior1)`, and `Complexity (prior 2)`, respectively. Based on the fit and two complexities, `BF-SEM` computes the Bayes factors under both prior distributions and displays them below `BFiu (prior 1)` and `BFiu (prior 2)` for H_i against H_u, and below `BFic (prior 1)` and `BFic (prior 2)` for H_i against H_{i_c}. We omit the results for hypothesis 2 and hypothesis 3, because they have the same form as hypothesis 1. Finally, the PMPs is printed, which can be obtained based on the results of the Bayes factors above. For each hypothesis, its PMPs under two prior distributions are written in the line below `PMP (prior1)` and `PMP (prior2)`.

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